

# Tutorial 12

## Advanced Graph Theory

### Planarity

13<sup>th</sup> November 2014

1. (Contd from Tut. 11)

Edges in a plane graph  $G$  form a cycle in  $G$  if and only if the corresponding dual edges form a bond in  $G^*$ .

Using this result, prove that a set of edges in a connected plane graph  $G$  forms a spanning tree of  $G$  if and only if the duals of the remaining edges form a spanning tree of  $G^*$ .

2. Prove that every 3-connected graph with at least six vertices that contains a subdivision of  $K_5$  also contains a subdivision of  $K_{3,3}$ .
3. Let  $H$  be a graph with maximum degree at most 3. Prove that a graph  $G$  contains a subdivision of  $H$  if and only if  $G$  contains a subgraph contractible to  $H$ .